Chiral perturbation theory for the Wilson lattice action*

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Chiral perturbation theory (χPT) is an important tool for extracting quantitative information from lattice simulations of QCD. The reason for this is that it is impractical to have dynamical quarks in simulations that are as light as the up and down quarks, and χPT is needed for a controlled, systematic extrapolation in the quark masses. Since χPT describes *continuum* QCD at low-energies, its application in numerical simulations is possible only after extrapolating lattice data to the continuum limit where the lattice spacing, a, vanishes. In this paper, we study the behavior of the Wilson lattice action close to the continuum by incorporating $O(a)$ effects in a reformulation of χPT. A similar approach was first taken in Ref. [1] to investigate the phase diagram for Wilson fermions in two-flavor QCD.

The quark mass matrix (considering only the 2 or 3 lightest quarks) has a special role in QCD — it parameterizes the explicit breaking of the axial symmetries. As a result, the light quark masses appear explicitly in the low-energy effective field theory (EFT). In this paper, we exploit the fact that for the Wilson lattice action there is another independent symmetry breaking parameter, linear in a [1, 2]. To $O(a)$ this is the only discretization effect, and thus a generalization of the chiral Lagrangian can be written which includes all terms linear in a. The extension to partially quenched (PQ) theory is also described.

We construct a low-energy EFT of the Wilson lattice action (WχPT) close to the continuum. The theory extends χPT, and the perturbative framework is described in terms of two small parameters — the quark mass m_q and the lattice spacing a. The Gasser-Leutwyler chiral Lagrangian through $O(p^4)$ in χPT is modified to incorporate all linear dependence on a.

This theory was applied to calculate light meson masses and decay constants. The resulting expressions capture all the linear dependence on a as well as non-trivial logarithms that entangle a and m_q. In the simplest sense, the expressions for the mass and decay constant in WχPT can be used to aid in taking the continuum limit. A test of these formulae would be to check whether they describe the a dependence better than naive extrapolations. A useful application of this theory is the determination of the Gasser-Leutwyler coefficients of ordinary χPT from lattice simulations at small but finite a.

What about higher orders in a? At order a^2 the picture changes qualitatively. There are operators such as $\bar{\psi}\not{D}D_\mu D_\mu\psi$, that do not break the chiral symmetry. This means that a can no longer be associated only with symmetry breaking effects, and spurion analysis cannot be used to constrain the a^2 operators. Nevertheless, we might still expand in m_q and a simultaneously. At $O(a^2)$, there are several operators that are added to the Lagrangian, but they are all independent of the quark masses and do not contain derivatives. Consequently, the only correction to the meson masses at this order is an additional term of the form ωa^2, where ω is an unknown constant of mass-dimension 4. The expression for the decay constant does not receive *any* corrections at this order. This is because tree level contributions to the decay constant can only come from operators that contain derivatives.

References

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